Best Kalman estimate computed by

$$\dot{\hat{\underline{T}}} = \underline{A}\,\hat{\underline{T}} + \underline{K}(t)\left[y(t) - [0 \ 1 \ 0]\hat{\underline{T}}\right]$$

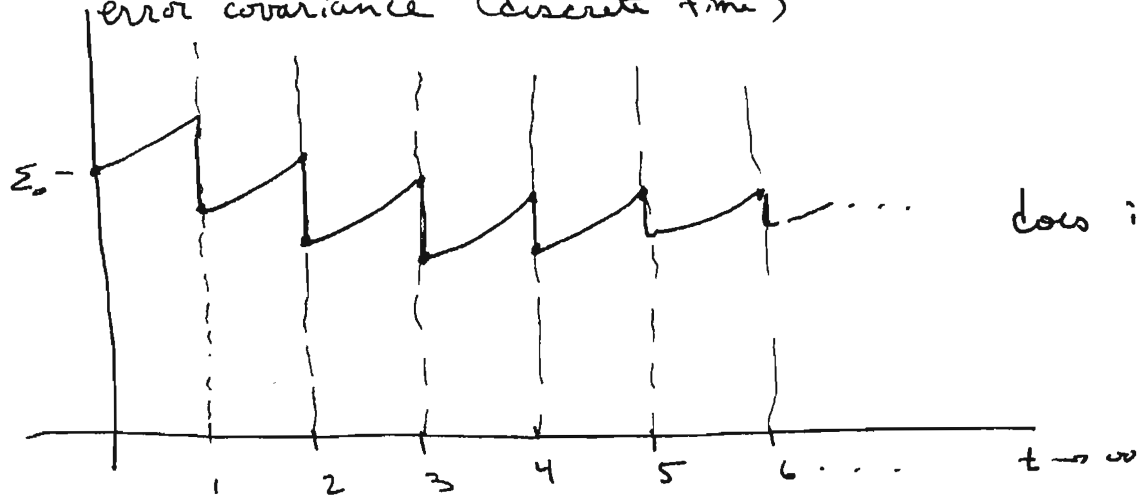
$$= \left[\underline{A} - \underline{K}(t)[0 \ 1 \ 0]\right]\hat{\underline{T}} + \underline{K}(t)\,y(t)$$

$$\underline{K}(t) = \underline{\Sigma}(t)\begin{bmatrix}0\\1\\0\end{bmatrix}\underline{R}^{-1} = \underline{\Sigma}(t)\begin{bmatrix}0\\1/R\\0\end{bmatrix}$$

$$\dot{\underline{\Sigma}}(t) = \underline{A}\,\underline{\Sigma}(t) + \underline{\Sigma}(t)\underline{A}^T + \underline{Q} - \underline{\Sigma}(t)\begin{bmatrix}0\\1\\0\end{bmatrix}\left(\tfrac{1}{R}\right)[0 \ 1 \ 0]\,\underline{\Sigma}(t)$$

$$= \underline{A}\,\underline{\Sigma}(t) + \underline{\Sigma}(t)\underline{A}^T + \underline{Q} - \underline{\Sigma}(t)\begin{bmatrix}0 & 0 & 0\\0 & 1/R & 0\\0 & 0 & 0\end{bmatrix}\underline{\Sigma}(t)$$

error covariance (discrete time)



does it converge?

convergent $\Longleftrightarrow$ observations contain some information about $x_t \; \forall \; t$.

We define the <u>information matrix</u> as

$$\mathcal{I}(t,t_0) \triangleq \int_{t_0}^{t} \underline{\Phi}^T(\tau,t)\,\underline{H}^T(\tau)\,\underline{R}^{-1}(\tau)\,\underline{H}(\tau)\,\underline{\Phi}(\tau,t)\,d\tau$$

- it is a measure of the information about $\underline{x}_t$ contained in the measurements

$$\mathcal{Y}_t = \{ \underline{z}(s) = \underline{H}(s)\underline{x}(s) + \underline{v}(s) \; ; \; t_0 \le s \le t \}$$

The <u>controllability</u> <u>matrix</u> is

$$\mathcal{C}(t,t_0) \triangleq \int_{t_0}^{t} \underline{\Phi}(t,\tau)\,\underline{G}(\tau)\,\underline{Q}(\tau)\,\underline{G}^T(\tau)\,\underline{\Phi}^T(t,\tau)\,d\tau$$

The <u>rank</u> of $\mathcal{I}$ is a measure of the number of states about which information is contained in $\mathcal{Y}_t$ (the dimension of the observable subspace). It is a function of the structure of the system (F,H), and of the degree of noise introduced by $\underline{v}(t)$.

The rank of $\mathcal{C}$ is the dimension of the subspace of the state space which is excited by the noise ~~b~~ process $\underline{w}(t)$. It is a function of the structure (F, G) and of the noise $\underline{w}(t)$.

The information matrix satisfies the differential equation

$$\frac{d\mathcal{I}}{dt} = -F^T(t)\,\mathcal{I}(t) - \mathcal{I}(t)\,F(t) + \underline{H}^T(t)\,\underline{R}^{-1}(t)\,\underline{H}(t)$$

Def: the system is completely observable (at t from $\mathcal{Y}_t$) iff $\mathcal{I}(t, t_0) > 0$.

Def: the system is completely controllable (at t ~~to~~ from $\underline{w}(t)$) iff $\mathcal{C}(t, t_0) > 0$.

Theorem: If the system is uniformly C.O. and C.C., then $\exists$ a ! positive definite solution $\underline{\Sigma}$ (steady-state) to the KBF equations.

[Def: Unif. c.o. (c.c.) if $\exists$ pos. constants $\sigma, \alpha, \beta \ni$

$$0 \leq \alpha \underline{I} \leq \underline{\mathcal{O}}(t, t-\sigma) \leq \beta \underline{I}$$

$$\left(0 < \alpha \underline{I} \leq \underline{\mathcal{C}}(t, t-\sigma) \leq \beta \underline{I} \right)$$

for all $t \geq t_0 + \sigma$.]

For the LTI system:

The steady-state solution exists and is unique iff $(F, G\sqrt{Q})$ is <u>stabilizable</u>

look up → and (F, H) is <u>observable</u>.

[controllable ⇒ stabilizable]

(F, G) controllable $\Leftrightarrow$ $[G \,|\, FG \,|\, F^2G \,|\, \cdots \,|\, F^{n-1}G]$ has rank $= n$.

(F, H) observable $\Leftrightarrow$ $[H^T \,|\, F^T H^T \,|\, F^{T^2} H^T \,|\, \cdots \,|\, F^{T^{n-1}} H^T]$ has rank $= n$.

- same result for discrete-time systems.

Algebraic Riccati Equations:

conts time:

$$0 = \underline{F}\,\underline{\Sigma} + \underline{\Sigma}\,\underline{F}^T + \underline{G}\,\underline{Q}\,\underline{G}^T - \underline{\Sigma}\,\underline{H}^T\underline{R}^{-1}\underline{H}\,\underline{\Sigma}$$

$$\underline{K} = \underline{\Sigma}\,\underline{H}^T\underline{R}^{-1}$$

$$\dot{\hat{\underline{x}}} = (\underline{F} - \underline{K}\,\underline{H})\hat{\underline{x}} + \underline{K}\,\underline{z}(t)$$

– use KBF or RICCND routines

discrete-time:

$$\underline{\Sigma} = \underline{F}\,\underline{\Sigma}\,\underline{F}^T + \underline{G}\,\underline{Q}\,\underline{G}^T - \underline{F}\,\underline{\Sigma}\,\underline{H}^T(\underline{H}\,\underline{\Sigma}\,\underline{H}^T + \underline{R})^{-1}\underline{H}\,\underline{\Sigma}\,\underline{F}^T$$

$$\underline{K} = \underline{\Sigma}\,\underline{H}^T(\underline{H}\,\underline{\Sigma}\,\underline{H}^T + \underline{R})^{-1}$$

$$\hat{\underline{x}}(t+1) = [\underline{F} - \underline{K}\,\underline{H}]\hat{\underline{x}}(t) + \underline{K}\,\underline{z}(t)$$

– use DKBF or RICDSD routines.