Digital PID Controllers

Conts - time PID:

$$e(t) \rightarrow \boxed{PID} \xrightarrow{m(t)}$$

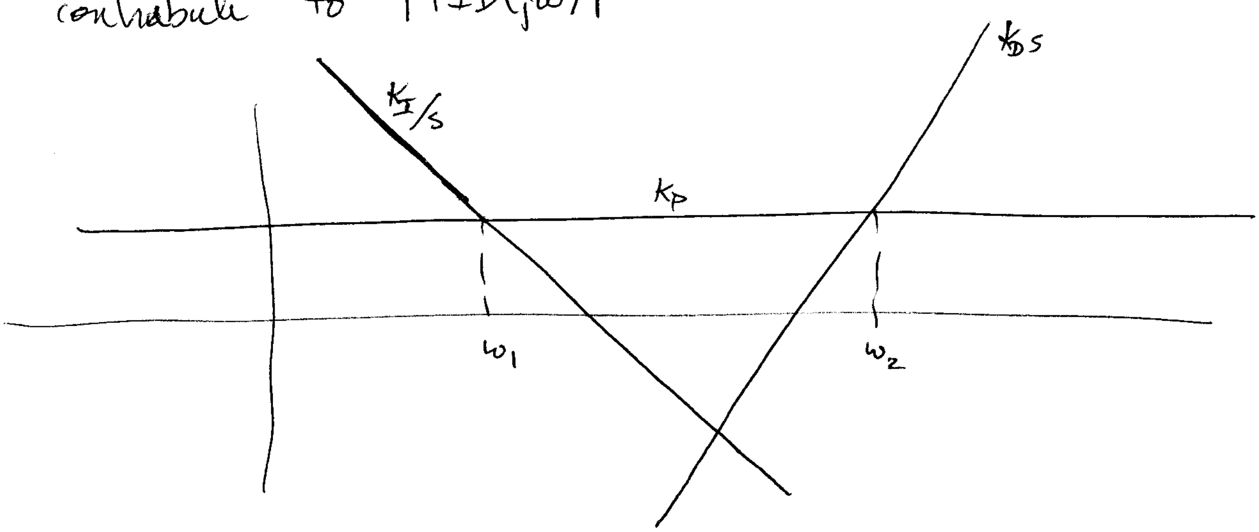
$$m(t) = \underbrace{K_p e(t)}_{\text{proportional}} + \underbrace{K_I \int e(t)dt}_{\text{integral}} + \underbrace{\frac{K_D de(t)}{dt}}_{\text{derivative}}$$

2 approaches:

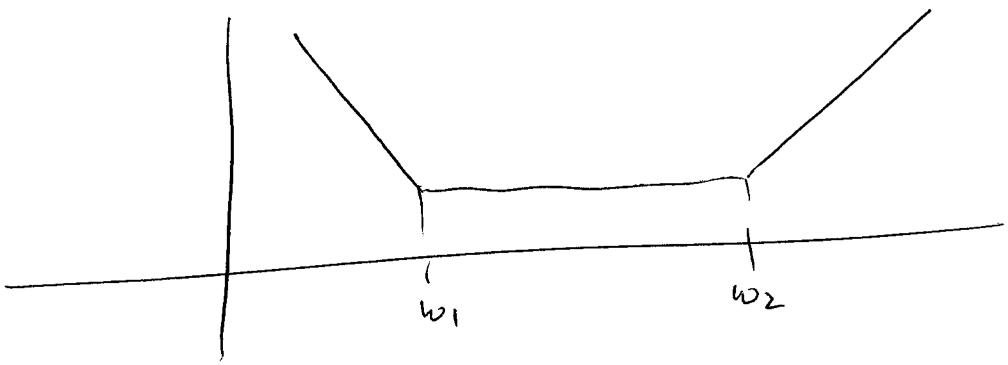
(1) Design in s plane & approximate $\int$ & $\frac{d}{dt}$

(2) Approximate $\int$ & $\frac{d}{dt}$ $\Rightarrow$ $D(z)$, then convert to w-domain, design, and implement coeffs on $D(z)$.

A quick review: the 3 components of PID contribute to $|PID(j\omega)|$:



So the approx magnitude of the PID controller is



Roughly, the choice of K_P determines the gain for $\omega_1 < \omega < \omega_2$.

K_I & K_P determine ω_1, and

K_D & K_P determine ω_2.

More precisely,

$$M(s) = K_P\, E(s) + \frac{K_I}{s} E(s) + K_D\, s\, E(s)$$

$$= \frac{K_D s^2 + K_P s + K_I}{s}$$

Note that this is <u>not</u> realizable b/c it is not proper — approximations must be made. The freq. response <u>must</u> roll off at high freq.

pole of PID : $s = 0$

zeros of PID : determined by $K_D s^2 + K_P s + K_I = 0$

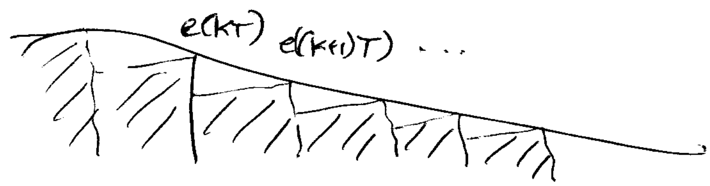
or $$s_{1,2} = -\frac{K_P}{2K_D} \pm \frac{\sqrt{K_P^2 - 4K_D K_I}}{2K_D}$$

where K_I, K_P, K_D are chosen so that $s_{1,2}$ are real $(K_P^2 > 4K_D K_I)$.

Method (1) - First, design cont's time PID (Chs. 7 & 9).

Then, approximate $\int$ & $\frac{d}{dt}$:

Euler's formula for approx. to integral :



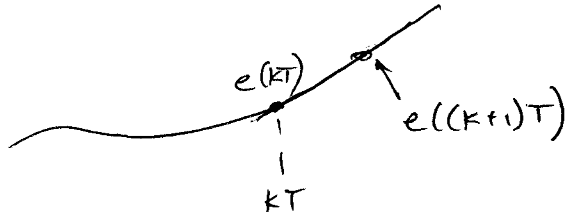
$$m_I((k+1)T) = m_I(kT) + Te((k+1)T)$$

or

$$z M_I(z) = M_I(z) + Tz E(z)$$

or $$\boxed{M(z) = \frac{Tz}{z-1} E(z)}$$

$\frac{d}{dt}$:



$$M_D((k+1)T) = \frac{e((k+1)T) - e(kT)}{T}$$

or

$$z M_D(z) = \frac{zE(z) - E(z)}{T}$$

or

$$\boxed{M_D(z) = \frac{z-1}{Tz} E(z)}$$

Substituting these approximations in to the PID:

$$M(z) = K_P E(z) + K_I \frac{Tz}{z-1} E(z) + K_D \frac{z-1}{Tz} E(z)$$

$$= \frac{K_P z(z-1) + K_I T z^2 + K_D (z-1)^2}{z(z-1)} E(z)$$

$$\doteq \underbrace{\frac{(K_P + K_I T + K_D) z^2 + (-K_P - 2K_D) z + (K_P + K_D)}{z^2 - z}}_{\text{digital PID designed using continuous-time PID's gains.}} E(z)$$

This is an approximation: Error will tend to make the c.l. system less stable.

See: Ex. 13.14.

Second approach: Start with

$$D(z) = K_P + K_I \frac{Tz}{z-1} + K_D \frac{z-1}{Tz}$$

convert loop transfer function to w-domain & select K_P, K_I, K_D.

This is also an approximation (because of Tustin). See Ex. 13.15.

A third approach:

(1) Design conts-time PID controller

(2) Convert to state-space realization

(3) Convert $\dot{x} = Ax + Bu$ to $x(k+1) = A_d x(k) + B_d u(k)$ using $e^{\begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} T} = \begin{bmatrix} A_d & B_d \\ 0 & I \end{bmatrix}$.

Root Locus design:

- Same as for cont. time, except for interpretation of the poles.
- Note that Ex 13.16 in book has an undesirable response due to pole introduced by the compensator.

Course Review

Broad strokes:

State-space design:

- one method: pole placement using Ackermann's formula
- design for controller (regulator) gain or estimator
- use both when states are not directly observable, but allow poles of only one to dominate
- observability & controllability - key concepts
- stability determined by eigenvalues of A.

The z-transform

- $z^{-1} \equiv$ one unit time delay.
- pole at $z = 1 \equiv$ pole at $s = 0$
- stability $\equiv$ poles inside open unit circle
- inverse transform using tables or long division
- state space realizations of system defined by z-Transform (1st & 2nd-d)

Sampled-data systems

- ideal sampler & the zero-order hold (ZOH)
- $*$-transform
- $*$ and product do <u>not</u> <u>commute</u>
- sampling + ZOH introduces phase lag & tends to destabilize system
- not all sampled data systems have transfer function representations
- discretization of system using $e^{\begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}T}$
- $\pm \frac{\omega_s}{2} \longleftrightarrow$ unit circle & repeating bands in s-plane
- Nyquist sampling rate - need for LP filters

Analysis

- Bilinear / Tustin Transformation
  - approximation breaks down as $|\omega| \to \frac{\omega_s}{2}$.
- Routh-Hurwitz or Jury's test for stability
- locations of poles for 1st & 2nd order responses.
- Root Locus: same except for root interpretation
- Nyquist - same
- Bode - Tustin & w-plane
- Steady-state accuracy.

Design

- phase lag — adj. s.s. perf.
- phase lead — adj. transient / h.f. perf.
- lead / lag (both!)
- digital PID

Things to remember

(1) sample + ZOH introduces phase lag (delay) & reduces stability margins

(2) Tustin / w-domain are approximate — may need to increase sampling rate

(3) Ensure no (little) signal content exists above $\frac{\omega_s}{2}$ before sampling!

(4) $*$-transform + s-domain harder than z

(5) z only provides information at sampling instants.

(6) Design choice — direct (z-domain, pole placement, etc) vs. w-domain + Tustin. (either works).